or she cannot avoid a difficult dilemma. Freiman names some individuals who have earned their trips abroad through ruthless enforcement of anti-Jewish measures at home. Should these emissaries of Soviet mathematics be boycotted or, on the contrary, do they deserve collegial treatment because mathematics must be kept apolitical, albeit on a unilateral basis? Is good will on one side sufficient to bring about harmony between two politically opposed partners?

This book serves as a warning to everyone. Discrimination has its own logic and inertia, particularly in the rationally organized societies which exist today. These considerations make the reading of Freiman's book very disquieting for someone who prefers to shun the ugly world around him and to take refuge in the abstract beauty of his chosen science.

THE EMERGENCE OF NUMBER. By John N. Crossley. Victoria, Australia (Upside Down A Book Company). 1980. 376 pp. Paper. A $14.50 + A $2.50 p. & P.

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In his Preface, the author stipulates: "In this book I try to trace the origins and early development of three kinds of number: the ordinary, natural counting numbers, complex and imaginary numbers and irrational numbers." One can quibble about this classification--complex numbers already include real, irrational numbers--but it apparently represents the order in which the author treats number, chapter by chapter. One may also feel disappointed that at least the elements of Cantor's transfinite numbers are omitted, since they provide such a well-documented example of how and why a new type of number originated. "My aim," states the author, "has not been to present a theory of the development of number but to seek out the genesis." Certainly the genesis of the transfinite numbers is spread before us in readily available documents.

Perhaps the first question that naturally occurs, in the light of these introductory remarks, is, "What is there to add to already known historical facts concerning the development of number?" Both anthropologists and historians of mathematics (e.g., V. G. Childe, E. B. Tylor, L. L. Conant, O. Neugebauer, K. Menninger) have given a multitude of case histories of primitive number systems and seem to have pretty well exhausted all available evidence concerning number origins. The author cites some of these, but in addition describes more recent developments; for example, the beautiful collection of Upper Paleolithic (ca. 20,000-7,000 B.C.) bones and stones on which occur groups of scratches evidencing the process of making permanent records via tallying, described with photographs in [Marshak 1972]; and in differentiating between counting words and quantification, the author draws a nice analogy between primitive number words and the basic color scheme discovered by Berlin and Kay [1969]. Another analogy, with what he terms "a primitive system," having the advantage that it is still evolving in electronics from its origin, consists of prefixes: *kilo-* for a 10^3 multiple (1795), *milli-* for a 10^{-3} fraction (1816), *mega-* for 10^6 (1868), *micro-* for 10^{-6} (1873), *giga-* for 10^9, *nano-* for 10^{-9} (1947), and so on (the author has seen *exa-* for 10^{15} and *peta-* for 10^{18} but judges they are not in common use yet).

In the second chapter, "The Natural Numbers--Historic Times," the distinction between the potential infinite and actual infinite is shown to be in both Aristotle and Euclid. The author repeats Euclid's proof of the theorem: "Prime numbers are more than an assigned multitude of prime numbers"; what Euclid did, given any finite set of primes (actually, he used three primes), was to show how to construct a prime not in the set. From this one can conclude by induction that the set of primes is infinite. Unfortunately most modern authors (who possibly do not read Euclid's proof) represent Euclid's theorem as stating that the number of primes is infinite, and as proving it by *reductio ad absurdum*. Hopefully the present work will help to correct this situation.

In the same chapter, the author gives a strong defense of Nicomachus' *Introductio arithmeticae*, a work that may not have been very original, but exerted a strong influence on arithmetic texts for centuries. As stated by Robbins and Karpinski in the D'Ooge translation,

> *Judged by the standards of the mathematician, Nichomachus cannot rank with the leaders of the science even as it was known in antiquity; estimated, however, by the number of his translators, scholiasts, commentators, and imitators, he is undoubtedly one of the most influential. From his own day until the sixteenth century, among the Greeks, the Latins, and the Arabs, there was scarcely a place where he was not honored as an arithmetician, or a time when learned men failed to regard his work as the basis of the science.*

Next comes a discussion of the evolution of mathematical induction, principally focusing on the work of Maurolico (1494-1575) which some (especially G. Vacca, 1909) consider the definitive origin of mathematical induction, but which others (notably H. Freudenthal, 1953) maintain to be only a type of indicated progressive proof, not actually induction. The 17th-century work of Wallis, closely approximating mathematical induction, and the definitive work of Pascal and Fermat (who used the equivalent method of finite descent) are described in detail. The chapter closes with a description of the work of Dedekind and Peano, done independently, on the formal axioms of the natural number system.

Two chapters (III, IV) titled "Latency" and "Revelation" respectively, are devoted to complex numbers. The "Latency" period covers Babylon, Euclid, the Hindus, Diophantus, the Arabs, and the early Renaissance Italians. From the standpoint of accomplishment, 40 pages seem too much to devote to "Latency," but apparently the author feels that complex numbers were lying, cocoon-like, in most of the writings of a numerical or algebraic nature during this period. Their ultimate "revelation" in the works of Bombelli and later authors is described in Chapter IV.

Chapter IV is replete with informative quotations from original sources, all translated into English and accompanied, in the Notes at the end of the book, by the originals, so that one can check the accuracy of all the translations. For example, the author gives Bombelli's description of how he acquired the material for the first draft of his *L'Algebra;* his subsequent discovery of Diophantus' work, which caused him to rewrite the algebra; his discovery of complex numbers which he manipulated by the rules already governing the "real" numbers known at the time. After discussing the work of Stevin and Viète, Crossley gives an unusually complete account of Harriot's work, including a copy of a page from Harriot's manuscript to demonstrate that, in contrast to the opinion of some historians, Harriot did accept complex numbers as solutions of equations. On the other hand, he concludes that Descartes "briefly considered" complex roots but did not use them. Curiously, he omits mention of Girard, who by his formulation of the fundamental theorem of algebra proved, to quote Struik, [1969], that he took complex roots "seriously." The chapter concludes with a translation of the Leibniz letter to Huygens (ca. March 1673) justifying Cardan's formula for solving cubic equations, and accepting complex numbers. Incidentally, in the part of the quotation on page 226, fourth line from the bottom, $\surd$ symbols are missing; however, the quote from the original French on page 347 is correct.

Chapter V, 50 pages long, is a slightly altered version of a paper published separately in the *Gazette of the Australian Mathematical Society,* Vol. 4 (1977), and Vol. 5 (1978), on irrational numbers. Here the reader is suddenly transported back to Greek and Egyptian mathematics, and then to Babylon--due, presumably, to the author's desire to give a strictly separate treatment of irrational numbers. It seems to the reviewer, however, that an inordinate amount of space is accorded to the mythology surrounding Thales and Pythagoras. Very little of this discussion has to do with irrationals, but rather with philosophy. Only in Section 9 does the author get down specifically to the discussion of irrationals, and then only to the approximations made in Babylonian arithmetic. In Section 10 the author returns to Pythagorean arithmetic in the use of "figurate numbers," as presented by Nichomachus. The concluding sections (11, 12) are entitled "Magnitudes and Ratios," and "The Introduction of Irrationals."

Chapter VI--"Real Numbers--Completion"--resumes with the work of Stifel (ca. 1487-1567), who maintained that irrationals were not "true" numbers, and that the circumference of a circle cannot be measured by a number (rational or irrational); and Maurolico, who did not consider magnitudes that are not describable by radical expressions. In Stevin's work on decimals, finite approximations are found, but not infinite decimals; noting that if the Euclidean algorithm does not terminate, the two magnitudes are incommensurable, he rightly observes that one cannot recognize incommensurability by such means. In Wallis' *Algebra* (1684) and Waring (1776) infinite series are developed, the latter giving a definition of convergence. After discussing W. R. Hamilton's ideas on defining ratios (adumbrative of Dedekind's work), the author concludes with the development of real numbers by Cantor and Dedekind. Regarding the (real) number system so defined, the author concludes (p. 309): "Everyone regards this system of numbers ... as being identifiable with the points or (*sic*) the line and Cantor's explicit introduction of an axiom [the 'Cantor axiom'] to justify that identification is generally forgotten." This is news to the reviewer, who referred to it in [Wilder 1965, pp. 88 f., 150].

The most enjoyable feature of this book, in the reviewer's opinion, is the wealth of quotations from primary sources--both in translation and in the original languages. As the author states in his concluding paragraph, he has "tried not to present old ideas in modern words but to let these ideas speak for themselves"; and "the moral of this book for those who have read it is: Do not read this book but go and read the originals, then you will see how number has emerged."

As may be expected in a first printing, the book contains a number of misprints. For example, on page 74 the explanation of the Chaap Wuurong expression for 90 is obviously incorrect; on page 221, footnotes 3 and 4 seem not to be indicated; on page 252, line 9, the reference to Heath's *Euclid* should be to his *History of Greek Mathematics* (that is, [91]); the reference to "Fig. 2" on page 279 is confusing, since figures are not numbered; on page 308, the definition cited here as "V. 5" in *Euclid* is "V. 4" in the Dover edition; on page 317, the Hilbert reference should be [95], not [93]; on page 356, note 2 of page 61, should read "See page 36, n. 1."

The book is supplied with copious notes, a long bibliography, and an index. It can serve as useful reading for students of the history of mathematics, as well as a source book for the scholar.

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PROLEGOMENA ALLE "VITE DEI MATEMATICI" DI BERNARDINO BALDI (1587-1596) (Accademia Polacca delle Scienze. Biblioteca e Centro di Studi a Roma: Conferenze e Studi 71). By Bronislaw Bilinski. Wroclaw, 1977. 135 pp. Zloty 38.

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Since the present reviewer and Professor Bilinski independently rediscovered Baldi's previously lost *Lives of the Mathematicians* in 1972-1973, interest in this landmark in the historiography of mathematics has quickened, and it may not be long before an edition and translations of the 200-odd lives are available; only a third of them have been edited to date, none of them in English translation. (Bilinski, p. 65, counts 202 lives and claims that 76 have been published to date; I count 204, of which 59 appeared.)

The *Prolegomena* offer a very useful description and listing of the rediscovered lives, together with an account of the history of the codices and notes on Baldi himself. Since Bilinski